Extensions of Lyapunov's ideas in the algebraic approximation of attractors

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We consider polynomial systems of the form

$$\dot{x} = f(x) \tag{1}$$

on $\mathbb{K}^n$, where $f \in \mathbb{K}^n[x_1, \ldots, x_n]$ is a polynomial and $\mathbb{K}$ denotes the field of real and complex numbers, respectively. It is assumed that the vector field (1) is complete and has constant divergence. It is well-known that the Lorenz equations and the Bloch equations which arise in nonlinear optics are examples of polynomial systems. In the contribution we present approximation procedures for attractors of (1) based on algebraic arguments. This means in the real case that semialgebraic sets of $\mathbb{R}^n$ of the type $\{x|p(x) = 0\}$ and $\{x|p(x) > 0\}$, where $p : \mathbb{R}^n \to \mathbb{R}$ is a polynomial, can be taken as approximation of the attractor. Conditions for the existence of such approximation sets can be expressed in terms of the Lie-derivatives of a polynomial Lyapunov function, computed with respect to the vector field (1). In the case when (1) represents the real or complex Lorenz system, we derive explicit semialgebraic sets approximating the attractors. It will be also demonstrated how to use algebraic approximation sets of attractors for the solution of control and observation problems of (1) in the class of polynomials.